This Mathematics Professional Learning Tool created for UCDSB captures the work of four years of professional learning in mathematics including assessment, big ideas in mathematics, engaging learning experiences, fundamentals of mathematics, intentional teacher moves, purposeful planning, the process of using continua and varying the structure of the problem.  

Thank you to everyone involved in planning and facilitating this learning over the last four years.
Tips and Strategies

You MUST use Adobe Reader: Steps to make Adobe Reader your Default Reader

Search the Document: CTRL + F

If the document looks distorted in the online version, click “Open in” button
Purposeful Planning

5 Practices
- Building the Math Community
- Vision of the Math Learner
- Building the Math Team
- Curriculum
- Effective Guides for Mathematics Instruction
- Guided Math Groups
- Know Your Learner
- Lesson Planning / 5 Day Plans
- Math Mindset
- Math Processes
- Minilessons
- Models Tools and Representations
- Concreteness Fading (VCPS)
- Indigenous
- Spiraling your Curriculum

Assessment
- Monitoring
- Assessment Loop
- Documentation & Triangulation of Data
- Achievement Chart & Evaluation
- Growing Success
- Descriptive Feedback
- EQAO

Continua
- Addition and Subtraction Continuum
- Multiplication and Division Continuum
- Fractions Continua
- Subitizing Learning Trajectory (Continuum)
- Comparing Learning Trajectory (Continuum)
- The Process of Using the Continua

Structure of the Problem
- Addition and Subtraction
- Multiplication and Division

Big Ideas in Math
- Algebraic Reasoning
- Proportional Reasoning
- Spatial Reasoning
- Mathematical Processes
- Subitizing
- Fractions

Engaging Learning Experiences
- Number Talks and Number String
- Inuit Inquiry
- Health and Physical Education
- Worthwhile Mathematical Tasks
- EQAO
### Introduction: 5 Practices for Orchestrating Productive Mathematical Discussions

1. **Anticipating**
   - Do the problem yourself
   - What are students likely to produce?
   - Which problems will most likely be the most useful in addressing the mathematics?

2. **Monitoring**
   - Listen, observe, identify key strategies
   - Keep track of approaches
   - Ask questions of students to get them back on track or to think more deeply

3. **Selecting**
   - CRUCIAL STEP – what do you want to highlight?
   - Purposefully select those that will advance mathematical ideas

4. **Sequencing**
   - In what order do you want to present the student work samples?
   - Do you want the most common? Present misconceptions first?
   - How will students share their work? Draw on board? Put under doc cam?

5. **Connecting**
   - Craft questions to make the mathematics visible.
   - Compare and contrast 2 or 3 students’ work – what are the mathematical relationships?
   - What do parts of student’s work represent in the original problem? The solution? Work done in the past?

---

### The Case of David Crane Reading – In the 5 Practices to Effective Instructions

The Case of David Crane

The Case of David Crane with Samples

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### Five Practices... Book

1. **ANTICIPATING**: As many representations of thinking as possible. What might my 3 marker students do?
2. **MONITORING**: Continuum of Learning
3. **SELECTING**: Which ones will I use for consolidation?
4. **SEQUENCING**: What order will we discuss them?
5. **CONNECTING**: Connect the learning to the big ideas

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### Slides 24 - 31

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### Slides 31 - 40
Mathematical Mindset

Paper Folding cut-out uncovers connections between spatial reasoning, problem solving and mathematical reasoning.

This book is in your school.
Creating an Optimal Math Learning Environment

Building Your 60 Minute Math Block

Classroom Discourse

Making Thinking Visible: Productive Talk Moves

- Revoicing—repeating what the students have said and then asking them for clarification
- Repeating—Asking student to restate someone else’s reasoning in their own words
- Reasoning—Asking student to apply their own reasoning to someone else’s reasoning (Agree or disagree and why)
- Adding on—Prompting students for further participation
- Waiting—Using wait time

(Chapin, O’Connor & Anderson, 2009)

Document: Making Student’s Thinking Visible

Lucy West – Types of Discourse

Word Wall – Using Mathematical Discourse

Visit Dan’s Website
www.mathforlove.com
Guided Math Groups

RMS Math System Day 4
March 21st and 22nd, 2018

We prepare all students for a successful life.

Slide 34

EduCANS

Lessons & Supports
TIPS4Math
Classroom Practices
Home Support
Payling Attention to
RMS PL
Webcasts

Targeted Implementation Planning Supports For Math

TIPS4Math

Grade 1 to 8 Curriculum Expectations – Coded +

Grade/Course Overviews (Scope and Sequences)
Each overview of expectations is considered a lesson bundle. Each of these bundles go further in depth for each particular grade (see below - Grade/Course Supports).

Please note that the greyed out text in the expectations are not being addressed in that particular lesson bundle. They are addressed in a different one. All expectations are from the current curriculum. The codes used can be referenced in the Grades 1 to 8 Expectations (see above).

Picture books that can support your students' learning in math....

Work Station Learning Experiences

THINKING BLOCKS
Model and Solve Word Problems
Algebra

Mathies Tools Summary Document
Setting the conditions for guided groups – what must you do before you consider doing guided groups.

**Video 1:** Setting the Conditions for Guided Practice

**Video 2:** Setting the Conditions for Guided Practice
Curriculum Connections and Big Ideas

Algebraic and Functional Reasoning Continuum of Curriculum Expectations
Proportional Reasoning Continuum of Curriculum Expectations
Measurement Continuum of Curriculum Expectations
Mathematics Within the Four Frames of the Kindergarten Program
Patterning and Algebra Continuum of Curriculum Expectations

Kindergarten Program – Links to the Mathematical Processes
Grades 7, 8 and 9 Math Curriculum Continuum
Data Management and Probability Continuum of Curriculum Expectations
Number Sense and Numeration Continuum of Curriculum Expectations
Geometry and Spatial Sense Continuum of Curriculum Expectations

The Ontario Curriculum
Grades 1-8

Mathematics

The Ontario Curriculum
Grades 9 and 10

Mathematics
Know your Learner

Student of Promise Case Study: Slide 28
Student of Promise Completed Sample Profile

Student with an LD Case Study: Slide 27
Student with an LD Completed Sample Profile

Student Consistently at Level 3 Case Study: Slide 29
Student Consistently at Level 3 Completed Sample Profile

Experience It: Math Issues

Have you ever wondered why it's so hard for some kids to work with numbers?
Lesson Planning / Five Day Plans

I Squared Day 2 – December 6th/8th

Slide 8

Planning Guide for Effective Mathematics Learning, Teaching, and Assessment over a 5-day cycle

Step 1
Know your learners, and
Know the learning

Step 2
Plan for responsive and differentiated assessment and instruction

Step 3
Facilitate, assess and respond in a co-learning environment

Step 4
Reflect on impact of assessment and instruction to determine next steps

Learning Experiences Planning Guide

Mapping Making Connections – Planning Document

Kindergarten Learning Experiences

Grade 2 – Measuring Area

Grade 4 – Multiplication

Grade 6 – Fractions

Grade 7 – Proportional Relationships using Percent, Ratio and Rate

Grade 8 – Linear Relations

Grade 9 – Linear Relations
Kindergarten Program – Links to the Mathematical Processes
Math Processes and the Achievement Chart Rubrics
Assessment for and as Learning with the Mathematical Processes
Achievement Chart Math Processes
Notes on How the Achievement Chart can support learning in Mathematics
Student work to go with Slides 13 and 14
EOCC Math Inquiry Project – Video and Documents for Slides 13 and 14
Problem Solving template #1
Problem Solving template #2
Problem Solving template #3
Problem Solving template #4
Problem Solving template #5
Problem Solving (Slide 19)
Problem Solving Success Criteria
Problem Solving Success Criteria Sample 1
Problem Solving Success Criteria Sample 2
**Mini-Lessons**

**Partial Products and Distribution**

Partial Products and Distribution: FFM Year 1 Day 3 Slides 75 – 77

**Math Learning Centre – Partial Products Generator**

**What to Look For**

Guides to Effective Instruction in Mathematics

Guides to Effective Instruction

Mini-Lessons found on Page 190

What to Look For. (copies in all schools)
Concreteness Fading (VCPS)

RMS Math System Day 3
January 10th and 12th

Thinking Deeper

Concreteness Fading Article
Skyscrapers Puzzles
V^3 – Visualize, Verbalize and Verify

Concreteness Fading Bookmark
Mathies Learning Tools

Back to Purposeful Planning
Back to Intentional Teacher Moves

Concreteness Fading Continuum

Visual → Concrete → Pictorial → Symbolic

Learning maths the easy way with abacus
Minds on: Paper Folding Mystery

• Observers will be walking around practicing pedagogical documentation while you work on this task
• **Visualize:**
• **Verbalize:**
• **Verify:**

Instructions for the Paper Folding Task

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Reflecting Back: Moving Forward

• What constitutes effective professional learning?
• **Visualize**
• **Verbalize** your ideas with a partner.
• Share with teammates. Sort ideas and classify.

---

Minds On: Visualize

Pick three consecutive numbers. Multiply the first number by the third number. Square the second number.
Models, Tools and Representations

Paying Attention to Fractions, Monograph

Set, Linear, Area and Volume Models (Slides 9 – 18, 34 – 40)

Set / Discrete Models:
In a set model, a collection of items represents the whole amount. Subsets of the whole make up the fractional parts.
Sample Tools: A variety of materials can serve as set models: coloured tiles, toys, discs, pictures of items, etc.

Linear Models:
Sample Tools: Relational Rods, 5 and 10 Frames, Number lines (open and closed, double, etc.), Fraction Strips, Rekenreks, Linking Cubes, Colour Tiles, etc.

Area Models:
Sample Tools: Rekenreks (multiple rows), Arrays (open and closed), 5 and 10 Frames, Algebra tiles, Coloured Tiles, Linking Cubes, Base 10 blocks, Pattern Blocks, Geoboards, Geometer’s Sketch Pad, Glimos, Graph paper, etc.

Volume Models:
In a volume model, a three-dimensional figure represents the whole. The whole is divided into fractional regions that are occupied by space within the figure. Sample tools: pouring containers

Colour Tiles
- Represent numbers, totals and percentages. Explore patterns and measurement. Create geometric designs. Users can change colours, create mirror images, rotate and copy solutions.

Exploring Different Representations
- Explore the Pattern Rule, Algebraic, Pictorial, Story and Graphical Representations of a linear growing pattern. Make a change to one representation and see how this change affects the other representations.

Fraction/Strips
- Represent fractions by dragging pieces from the fraction tower into the workspace. Pieces can be placed in a line to form a train. Manipulate the pieces to compare and order fractions or to model fraction operations.

Money
- Represent various amounts using realistic images of Canadian coins and bills. Customize the money tray, drag money simply or in multiples of 2, 5, and 10, and annotate waypoints.

Notepad
- Write solutions, sketch diagrams, import pictures, create graphs, and utilize rulers, number lines, grids and shapes to record your mathematical thinking. Use one of four backgrounds: grid, geometric dot, tiled or blank.

Number Line
- Represent, compare, order and operate on whole numbers and decimals using vectors, hops, magnitude bars, number lines or points on the number line. This dynamic tool makes it easy to zoom in, pan and switch between horizontal and vertical number lines.

Partitioning Sets
- Partitioning sets allowing students to model 1/13 of a set of

mathies Learning Tools

Classroom Educator School Leader System Leader Professional Learning Facilitator Contact Us

Manipulatives EduGAINS

Interactive electronic supports

January 9th and 10th, 2018
Troy Maracle, Indigenous lead from HPEDSB reminds us that “we need to teach to the head and the heart, but we know we will have had an impact when we reach the heart. When we connect to the feeling our learning can go so much deeper.”
Why do we Spiral?

and apply throughout the year, regardless of the strand being studied. Teachers should ensure that students develop their ability to apply these processes in appropriate ways as they work towards meeting the expectations outlined in all the strands.

When developing their mathematics program and units of study from this document, teachers are expected to weave together related expectations from different strands, as well as the relevant mathematical process expectations, in order to create an overall program that integrates and balances concept development, skill acquisition, the use of processes, and applications.

Many of the expectations are accompanied by examples and/or sample problems, given in parentheses. These examples and sample problems are meant to illustrate the learning, the kind of skill, the depth of learning, and/or the level of complexity an example entails. The examples are intended as a guide for teachers rather than a mandatory list. Teachers do not have to address the full list of examples; rather, one or two examples from the list and focus also on closely related areas of learning. Similarly, teachers are not required to use the sample problems to supplement the sample problems into their lessons, or they may use other problems to the expectation. Teachers will notice that some of the sample problems requirements of the expectation at hand but also incorporate knowledge of expectations in other strands of the same grade.

Sample Spiralled Courses

Grades 1 – 8

Grade 9 Applied

Grade 10 Applied

Grade 12 College (MAP/MCT)

Kyle Pearce has videos, workbooks and ideas on how to spiral. Teacher Desmos
Building the Math Community

**Vision of the Math Learner**

Yes, I Can! Monograph
NTIP 2018 - Slides 24 – 27
RMS Year 1 Day 1 - Slides 18 – 20
RMS Year 1 Day 5 - Slide 8

Pedagogical Systems – Vision of the Math Learner

Pedagogical Systems – Look Fors

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**Building the Math Team**

RMS Year 1 Day 1 - Slides 9, 22 – 26, 28
RMS Year 1 Day 5 - Slides 6 -7 and 24
I² Day 3 - Slides 4 and 5

Team Building Challenge: Instructions in Notes Slide 25

Group Norms Handout

Norms in the Mathematical Classroom

PPM 159 – Collaborative Professionalism

Setting the Table for Collaborative Professionalism

Video: Building Networked Learning Communities – Steven Katz

Guiding Questions for Discussion Around Learning
**Assessment**

**Monitoring**

**Assessment Loop**
- Learning Goals and Success Criteria
- Monitoring Learning and Setting Goals
- Eliciting Student Thinking and Learning
- Generating Descriptive Feedback
- Engaging in Peer and Self-Assessment

**Documentation**

**Achievement Chart & Evaluation**

**Growing Success**

**Classroom Discourse – Math Talk**

**Integrating Assessment & Instruction in to 3 Part Lesson**

**Triangulation of Data**

**Noticing and Naming**

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**Fundamentals of Mathematics**

**Intentional Teacher Moves**

**Purposeful Planning**

- 5 Practices
- Curriculum
- Effective Guides to Mathematics Instruction
- Know Your Learner
- Lesson Planning / 5 Day Plans
- Minilessons

**Continua**

- Addition and Subtraction Continuum
- Multiplication and Division Continuum
- Fractions Continua
- Subitizing Learning Trajectory (Continuum)
- Comparing Learning Trajectory (Continuum)
- The Process of Using the Continua

**Structure of the Problem**

- Addition and Subtraction
- Multiplication and Division

**Big Ideas in Math**

- Algebraic Reasoning
- Proportional Reasoning
- Spatial Reasoning
- Mathematical Processes
- Subitizing
- Fractions

**Engaging Learning Experiences**

- Number Talks and Number String
- Inuit Inquiry
- Health and Physical Education
- Worthwhile Mathematical Tasks
- EQAO

---

**Fundamentals of Mathematics**

- Fundamentals of Mathematics
  - The Learning Exchange
  - Fundamental Math Concepts and Skills
  - Teacher Guide

**Intentional Teacher Moves**

- Guided Math
- Games
- Concreteness Fading (VCPS)
- Accountable Math talk
Connections between Instruction and Assessment – See notes on Slide 2

Slides 6-7 and 39-53
### Grade 8 Curriculum:
To represent the multiplication and division of fractions in a variety of ways.

<table>
<thead>
<tr>
<th>Learning Goal:</th>
<th>Success Criteria:</th>
</tr>
</thead>
<tbody>
<tr>
<td>We are learning to represent the product of two fractions using an area model.</td>
<td>To be co-constructed</td>
</tr>
</tbody>
</table>

[Slides 14 – 18 and 21 - 25]
What tools do you use when assessing - what tools to you use when evaluating?
Research statistics on educator responses - Information in the slides 9 - 15

Mentimeter Tool - a great way to capture educator or student responses and see the results in real time

**Building Capacity in Assessment and Mathematics**

Building Capacity in Assessment in Mathematics Yr 3 Day 2-
Slides 21 – 29

13 Samples: Five Types of Assessment Tools: Marking Scheme, Rubric, Checklist, Rating Scales, and Anecdotal Records
Minds On! Paying Attention

...what teachers attend to as they teach is highly consequential. Given that, the next logical questions become: How and why does it matter, and what can be done about it? (A. Schoenfeld, p. 224)

Monkey Business Video

Count how many times the players wearing white pass the ball.
There are seven videos embedded in the slideshow:

- CBS: Pedagogical Documentation Revisited Monograph
- Edutopia Article: Five Classroom Tools to Measure Student Learning
- Edutopia Article: The Art of Reflection

**Slides 18 – 23 and 25 – 26**
- Primary (7 videos)
- Junior (9 videos)
- Intermediate (6 videos)
- Secondary (8 videos)
- Gathering Valid and Reliable Evidence of Learning to Determine a Grade Video Library

**Making Connections Planning Template**
Eliciting Student Thinking and Learning

Capacity Building Series

Cultivating Classroom Discourse to Reveal Student Thinking

Effective Practice Includes Discourse EOSDN, April 2016
Lucy West lucy@lucywestpd.com
Metamorphosis website: www.lucywestpd.com

Making Student Thinking Visible
Research for Better Teaching
Empowering Sustainable School Improvement

Being Responsive to Student Thinking

When educators work with a partner or in small groups to examine and annotate student work, it allows for student thinking to be unpacked – their understandings, strategies and transitional conceptions. Here, student thinking from a Grade 3 classroom, a Grade 7/8 classroom and a Grade 9 classroom is shared for educators to use in their own professional learning, individually or school-wide.
Generating Descriptive Feedback

Minds On! Feedback

- What is the teacher doing?
- What is/are the students doing?
- What kind of feedback might they be receiving?
- How might we document this feedback?
- What are the challenges or barriers involved in this way of giving feedback?

Descriptive Feedback: Assessment for Learning Video Series Viewing Guide

Descriptive Feedback: Video Library

Descriptive Feedback

Read the article titled “Descriptive Feedback” and code it as follows:

✓ This affirms my thinking
! This surprises me
? I’m wondering about

Growing Success Anticipation Guide

Anticipation Guide with Descriptions

Anticipation Guide Responses

The content of the descriptive feedback should be rooted in the success criteria.

Slides 47 - 51

Growing Success

Anticipation Guide

Anticipation Guide with Descriptions

Anticipation Guide Responses
Learning Goals and Success Criteria

Slides 24 – 26 and 32 – 46
Slides take you through moving from curriculum to setting goals, then creating success criteria, and determining tasks.

Making Connections Planning Template

Learning Goals and Success Criteria

Co-constructed Success Criteria on Slide 30

Unpacking the Learning Goal

Mindomo

What’s important when writing a learning goal

- Learning comes from identified curriculum
- Translating curriculum expectations into language that students will understand
- Define success criteria that goes with the learning goal
- First person (I can.. We can); ownership of the goal

Slides 34 and 35

Slides 11-15 and 19 - 26

Slides 20 - 23
1. From Viewing Guide: Learning Goals and Success Criteria
3. From How to Give Effective Feedback to Your Students Choosing feedback Content (task, process, self-regulation, person)
4. From How to Give Effective Feedback to Your Students: The Role of Feedback in Effective Formative Assessment
5. From How to Give Effective Feedback to Your Students: The Role of Feedback in the Regulation on Learning
6. From How to Give Effective Feedback to Your Students: What does feedback as an episode of learning look like? Samples at each division
7. From How to Give Effective Feedback to Your Students: Samples (math solutions)
8. From Choice Words: The Importance of Noticing and Naming
9. Asking Effective Questions Monograph
10. Mathematical processes – sample questions and feedback
11. From Advancing Formative Assessment in Every Classroom - Chpt. 2: Leveling the Playing Field: Sharing Learning Targets and Criteria for Success
12. From Advancing Formative Assessment in Every Classroom - Chpt. 3: Shifting from Correcting to Informing: Feedback that Feed Forward
Student Self Assessment

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Andrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is SSA?</td>
<td>p. 92 and first ½ of 93</td>
</tr>
<tr>
<td>Why should we teach students to do it?</td>
<td>(p. 93 and first ½ of 94 - research on self-assessment)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group 2</th>
<th>McMillan &amp; Hearn</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is student Self-Assessment?</td>
<td>The Self-Assessment Process</td>
</tr>
<tr>
<td>p. 40 and 41</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group 3</th>
<th>Moss &amp; Brookhart</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is SSA?</td>
<td></td>
</tr>
<tr>
<td>What common misconceptions are held about SSA?</td>
<td></td>
</tr>
<tr>
<td>How does SSA affect student learning and achievement?</td>
<td></td>
</tr>
<tr>
<td>What is the motivation connection?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group 4</th>
<th>McMillan &amp; Hearn</th>
</tr>
</thead>
<tbody>
<tr>
<td>A theoretical rationale for enhancing Self-Assessment</td>
<td>p. 42/44</td>
</tr>
</tbody>
</table>

Slides 16 to 18

Student Self-Assessment: The Key to Stronger Student Motivation and Higher Achievement

What is Student Self-Assessment?

Digital Documentation: Using Digital Technologies to Promote Language Assessment for the 21st century

Students: Source of Formative Assessment

Student Self-Assessment Organizer

Asking Effective Questions Monograph
Slide 39  Focus Student Monitoring Template
Slide 41  Educator Self Assessment Form

Slides 9 – 13  Guiding Questions for unpacking Student Learning

This slideshow shows examples of using forms to track student progress using success criteria.
Myths vs Realities

1. Diagnostics:
2. Success Criteria and the Design of Tasks:
3. Assessment tools:
4. Effective Feedback:
5. Student Work, Report Card Comments:
6. Achievement Chart Math Processes:
7. Triangulation Validity
8. Determining the Grade:
9. Is it possible?
10. Self and Peer Assessment:

Learning Experience: Getting to know...
- In your group discuss the question on the chart paper and record your current understandings.
- What do you know about Assessment for Learning?
- What do you know about Assessment as Learning?
- What do you know about Assessment of Learning?

Growing Success Anticipation Guide: Template
Growing Success Anticipation Guide: Explanations
Growing Success Anticipation Guide: Responses
## Growing Success Myths vs Realities

<table>
<thead>
<tr>
<th>Step</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Diagnostics</td>
<td>How might we uncover what students already know or pinpoint possible mathematical misconceptions? (assessment for learning – use of diagnostic assessments)</td>
</tr>
<tr>
<td>2. Success Criteria &amp; Task Design</td>
<td>What are the implications of increasing the use of success criteria in everyday practice? How might this process impact the design of assessment tools, including rubrics?</td>
</tr>
<tr>
<td>3. Assessment Tools</td>
<td>How might the design and use of assessment of learning tools and strategies, including test and assignments, reflect key ideas of effective learning and teaching in mathematics?</td>
</tr>
<tr>
<td>4. Effective Feedback</td>
<td>What constitutes effective feedback in mathematics? How might descriptive feedback differ from evaluative feedback?</td>
</tr>
<tr>
<td>5. Student Work, Report Card Comments</td>
<td>What is important when designing meaningful report card comments? How might analysis of student work samples support designing report card comments?</td>
</tr>
<tr>
<td>6. Achievement Chart Math Processes</td>
<td>How might the categories of the Achievement Chart support learning in Mathematics? What connections exist between the mathematical processes and the achievement chart?</td>
</tr>
<tr>
<td>7. Triangulation Validity</td>
<td>How might triangulation of data increase validity of assessment of learning and evaluation?</td>
</tr>
<tr>
<td>8. Determining the Grade</td>
<td>How might we gather valid and reliable evidence of learning to inform educators’ professional judgment so that we can communicate effectively about student learning (including determining a grade)?</td>
</tr>
<tr>
<td>9. Is it Possible?</td>
<td>What key policy statements should be considered when aligning instructional practices to effective assessment strategies? (e.g., considering group work, homework, self-assessment, structure of rubrics, reflections on level 4)</td>
</tr>
<tr>
<td>10. Self and Peer Assessment</td>
<td>How might we build self and peer assessment skills with students?</td>
</tr>
</tbody>
</table>
Structure of the Problem

Adding and Subtracting

- RMS Year 2 Day 1 – Slides 18 - 25
- Blank Template
- Sorting Cards
- Completed Template
- Semantic Structure Explanations
- Strategies for How to Move Students from Semantic to Computational

Multiplication and Division

- FFM Year 1 Day 3 – Slides 24 – 31, 43 – 46
- PQM Cards
- Partitive/Quotative/Multiplicative - Blank Template
- Partitive/Quotative/Multiplicative - Completed Template
- Multiplication and Division Situations – Going Deeper Blank
- Multiplication and Division Situations – Going Deeper Completed
- Multiplication Situations
- Practice Questions – Slides 15 – 19
- Practice Questions with Fractions – Slides 86 - 90

Purposeful Planning

- 5 Practices
- Curriculum
- Effective Guides to Mathematics Instruction
- Know Your Learner
- Lesson Planning / 5 Day Plans
- Minilessons

Fundamentals of Mathematics

- Math The Learning Exchange
- Fundamental Math Concepts and Skills
- Teacher Guide

Other Resources

- RMS Year 2 Day 1 – Slides 18 – 25, 30 – 31
- 4 Part Series on Operation Sense

Welcome to this four-part series on Operation Sense. In this series we explore:
- the difference between computational strategies and operation sense;
- the meaning of and the relationships between the four operations;
- the ways in which operation sense and computational fluency develop over time;
- the importance of representations and models in understanding the operations.

Trajectory of Operation Sense
Engaging Learning Experiences / Worthwhile Math Tasks

**Number Talks and Number Strings**

**Inuit Inquiry**

**Indigenous**

**Health and Physical Education**

**Coding**

**Worthwhile Mathematical Tasks**

**Open vs Closed Problems (Slides 32 - 57)**

**3 Act Math Tasks**

**Useful Print Resources**

**Books in Schools**

**Games**

**Engaging Online Resources used at RMS / FFM**

**EQAO**

<table>
<thead>
<tr>
<th>Intentional Teacher Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guided Math</td>
</tr>
<tr>
<td>Games</td>
</tr>
<tr>
<td>Concreteness Fading (VCPS)</td>
</tr>
<tr>
<td>Accountable Math talk</td>
</tr>
</tbody>
</table>
Useful Print Resources

Don’t forget all the books in your school
## Engaging Online Resources used at RMS / FFM

### Ministry of Education Websites: Be sure to check these out

- Math.thelearningexchange.ca
- Edugains
- Mathies
- Tips4Math
- Mentimeter
- youcubed®
- The MATH LEARNING CENTER
- Which One Doesn’t Belong!
- EduGAINS
- Visual Patterns
- Math Designs
- Eyes on Math
- 3 ACT MATH
- math4love
- 180
- polyup
- Problem of the Week
- inside $+ \times = +$ mathematics
- Metamorphosis
- Teaching Learning Communities
- Math Playground
- [LT]$^2$
- Idao•Learning Disabilities Association of Ontario
- Teacher Desmos
Games

YouCubed Math Games

“What game, for what purpose, for which child, at this moment in time?”

Games

Games那 involve Making 10

<table>
<thead>
<tr>
<th>Game</th>
<th>Promotions</th>
<th>Where Referred</th>
<th>Materials</th>
<th>Group Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Fish 10</td>
<td>• strategy of using the free choice • key idea of the part-whole relationship • adding and subtracting</td>
<td>Slide 7-8 (page 101) Slide 7-1 (page 102) Slide 7-2 (page 103) Slide 7-3 (page 104)</td>
<td>a standard deck of playing cards with the jack, queen, and king removed and aces as 1s</td>
<td>2 or 4 players</td>
</tr>
<tr>
<td>Mix Up 10 Concentration</td>
<td>• strategy of using the free choice • key idea of the part-whole relationship • adding and subtracting</td>
<td>Slide 8-3 (page 101) Slide 8-4 (page 102) Slide 8-5 (page 103) Slide 8-6 (page 104)</td>
<td>a standard deck of playing cards with the jack, queen, and king removed and aces as 1s</td>
<td>3 players</td>
</tr>
<tr>
<td>Steal the Band</td>
<td>• strategy of using the free choice • key idea of the part-whole relationship • adding and subtracting</td>
<td>Slide 9 (page 101) Slide 9-1 (page 102) Slide 9-2 (page 103) Slide 9-3 (page 104)</td>
<td>a standard deck of playing cards with the jack, queen, and king removed and aces as 1s</td>
<td>2 or 4 players</td>
</tr>
<tr>
<td>Mix Up 10 and Add to the 10</td>
<td>• strategy of using the free choice • key idea of the part-whole relationship • adding and subtracting</td>
<td>Slide 10-4 (page 101) Slide 10-5 (page 102) Slide 10-6 (page 103) Slide 10-7 (page 104)</td>
<td>a standard deck of playing cards with the jack, queen, and king removed and aces as 1s</td>
<td>3 players</td>
</tr>
</tbody>
</table>

Summary of games in What to Look For

Fill the Tower Alternate Versions

Tug of War
Engaging Learning Experiences (ELE) / Worthwhile Math Tasks

What Makes an ELE
- What Constitutes an Effective Task Brainstorming Template
- Worthwhile Math Tasks? Questions to Ask Yourself
- Models for Determining if a Task is Worthwhile – Slides 8 - 16

ELE Components Explained (SIPSAW)
- Slides 23 – 48 and Placemat for Engaging Learning Experiences
- Components Explained Handout
- Starting Small and Planning Handout

Algebraic Reasoning
- Slide 23
- Paul’s Quilt and Paul’s Quilt Continuum
- Matchstick Task and Matchstick Task Continuum
- Wading Pool Task and Wading Pool Task Continuum
- Caterpillar Task and Caterpillar Task Continuum
- The Turkey Problem – Slide 16 and Transcript
- Secret Pattern Rule – Slides 24 – 31 and Document
- Algebraic and Functional Reasoning Continuum of Curriculum Expectations

Fractions
- Ribbon Task – Slides 7 – 11
- Ribbon Task – Student Video #1, Video #2
- RMS Year 2 Day 3 – Slides 22 – 42
- Fractions Matching Activity and Fraction Flag Task
- Lemonade Problem and Student Solutions
- Primary Fraction Experiences
- Junior Fraction Experiences
- Intermediate Fraction Experiences
- Fractions Using the Assessment – Slides 18 – 29
- Student Video of Chocolate Bar Task
- The Learning Exchange Videos on Visuals to support on Composing and Decomposing Numbers

Problems By Strand/Type
- Primary Open Response EQAO
- Junior Open Response EQAO
- Grade 9 EQAO
- Data Management and Probability
- Geometry and Spatial Sense
- Patterning and Algebra
- Number Sense and Numeration
- Measurement

Websites:
- Estimation 180 – Cheese Ball Problem Slide 7
- Solve me Mobiles – Slides 13 – 19
- Graham Fletcher – 3 Act Tasks
- Tap Into Teen Minds – 3 Act Task
- Mr. Orr is a Geek – 3 Act Tasks
Inuksuk, Inuit Games and Igloo Learning Experiences (K, 1 - 2, 3 - 4, 5 - 6) for a total of twelve lessons with slide decks and cultural teachings.

Inuit Inquiry Slide Deck explaining the Inquiry Process for this inquiry.
All students receive at least twenty minutes of moderate to vigorous physical activity during instructional time every school day.

Activities are adapted, as appropriate, to ensure that all students, including students with special education needs, can participate.

active kids learn better

physical activity at school is a win-win for students and teachers

Grades:
- 20% more likely to earn an A in math or English

Standardized Test Scores:
- 6% increase over 3 years

Just one physically active lesson creates:
- 13% increase in students' physical activity for the week
- 21% decrease in teachers' time managing behavior

PPM 138

TDSB H & PE lessons in every grade where mathematics is infused within the document
Number Talks

Engaging Students in Mathematical Thinking
NTIP Presentation – March 27th, 2018

Slides 14-22

Minds On – More Number Talks
Which One Doesn’t Belong:
www.wodb.ca

Slides 2 - 4

Number Strings
K-2: Skip counting with Rekenreks
Grade 3-4: Halving and doubling
Grades 7-10: Multiplying decimals using arrays.

Slide 8

Over the break, I got a new Fitbit and started tracking the distance I travelled.
- On Monday, I walked to school and back, and my Fitbit tracked 6 km from home to school.
- On Tuesday, I walked to school and then after school I walked directly to the outdoor rink.
  After my fun at the rink, I walked home. The distance from the rink to home is 1.5 times the distance to school.
- On Wednesday I walked to school. After school, I walked 2/3 of the way home to get to Makepeace at the library and then my mom picked me up after and drove me home.
- How far did I walk each day?
- What was the total distance I walked?

Slide 29

Talking Points for Number String on Slide 29

20 Days of Number Sense & Rich Math Talk
#20DaysNS

Slide 29

Activation: Visual Number Talk
How did you see this image?

Slide 8

Jo Boaler leading the 7 dot number talk

Slide 6 and 7

Jo Boaler leading the 7 dot number talk

Steve Wyborny,
Cube Conversations
Addition and Subtraction Continuum
Multiplication and Division Continuum
Fractions Continua
Subitizing Learning Trajectory (Continuum)
Comparing Learning Trajectory (Continuum)
Counting Principles
Grades 7, 8 and 9 Math Curriculum Continuum

Process of Using the Continua

- Addition/Subtraction & Multiplication/Division
  - Primary
  - Junior
  - Intermediate
- Subitizing

Practice the Process of Using the Continuum

- Anticipate (do the math using different strategies)
- Notice and Name Student Work
- Ask Effective Questions
- Provide Descriptive Feedback
- Place on Continuum
- Determine Key Ideas (where on the band?)
- Intentional Teacher Moves
Addition and Subtraction Continuum from What to Look For

Process of Using the Continuum

Slides 35 – 51
Slides 23 - 55
Primary Videos
Junior Videos
Intermediate Videos

Key Ideas

Slides 16 – 17
Matching Game Cards
Summary Sheet
Commutative Property – Slide 34

Direct Modeling Vs Tracking – see Information on Slides 32 and 33

Strategies and Samples of Student Work

Slides 22 – 31
Addition and Subtraction Continuum
Blank Look Fors
Sorting Cards with Strategy Titles
Sorting Cards without Strategy Titles
Sorting Cards of Sample Student Work
Primary Completed Looks Fors Continuum with Student Samples
Junior Completed Looks Fors Continuum with Student Samples
Intermediate Completed Looks Fors Continuum with Student Samples
Slides 20 – 22
Primary Sorting Cards
Primary Completed Representations with Some Errors
Primary Looks Fors and Strategies with Sample Student Work
Junior Looks Fors and Strategies with Sample Student Work
Junior Completed Representations Sorting Cards
Intermediate Sorting Cards
Intermediate Look Fors for Strategies and Samples of Student Work
Intermediate Completed Look Fors and Representations
Intermediate Completed Representations with Some Errors
### Multiplication and Division Continuum from What to Look For

**Alex Lawson’s What to Look For Continuum of Numeracy Development**

**Multiplication and Division**

<table>
<thead>
<tr>
<th><strong>FOCUS STUDENT</strong></th>
<th><strong>DIRECT MODELLING &amp; COUNTING</strong></th>
<th><strong>COUNTING MORE EFFICIENTLY &amp; TRACKING</strong></th>
<th><strong>WORKING WITH NUMBERS</strong></th>
<th><strong>PROFICIENCY</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Substituting</td>
<td>Counting Rhythmically</td>
<td>Using $10x$</td>
<td>Using Automatic Retrieval</td>
</tr>
<tr>
<td></td>
<td>Modelling Composite Units and Counting by Direct (and if Division, Re-counting)</td>
<td>Counting All Grouping by Composite Counting Groups</td>
<td>Using $1x$</td>
<td>Using Strategic Efficient Methods</td>
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<tr>
<td></td>
<td>Counting All Grouping by Composite Counting Groups</td>
<td>Using Trial and Error</td>
<td>Using Repeated Addition</td>
<td>Using Partial Products</td>
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<tr>
<td></td>
<td>Representing Empty Groups and Non-Sharing</td>
<td>Using Repeated Subtraction</td>
<td>Doubling</td>
<td>Doubling and Halving</td>
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<tr>
<td></td>
<td>Carrying</td>
<td></td>
<td>Using a Ratio Table</td>
<td>Using Alternative or Standard Algorithms</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>KEY IDEAS</strong></td>
<td><strong>Process of Using the Continuum</strong></td>
<td><strong>Strategies and Samples of Student Work</strong></td>
<td><strong>Primary Videos</strong></td>
<td><strong>Junior Videos</strong></td>
</tr>
<tr>
<td>Slides 35 – 51</td>
<td>Slides 34 – 36</td>
<td>Slides 24 - 25</td>
<td>Primary Videos</td>
<td>Junior Videos</td>
</tr>
<tr>
<td>Slides 23 - 55</td>
<td>Key Ideas Developing Matching Games</td>
<td>Blank Look Fors Continuum</td>
<td>Intermediate Videos</td>
<td></td>
</tr>
<tr>
<td><strong>Intermediate Videos</strong></td>
<td>Summary Sheet</td>
<td>Look Fors Sorting Cards – With Headers</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Look Fors Sorting Cards – Without Headers</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Look Fors Solution – all Strategies</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Rhythmic Counting Slide 47

**Rhythmic Counting Video**
Partitive Fraction Strategies Continuum

<table>
<thead>
<tr>
<th>Strategy</th>
<th>What it looks like</th>
<th>Related key ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>No coordination between sharers and shares</td>
<td>![Image of a fraction divided among sharers]</td>
<td>It is not yet developed. Questions to consider:</td>
</tr>
<tr>
<td>Repeated halving: coordination at the end</td>
<td></td>
<td>- Why did you cut the last granola bar into 6 pieces?</td>
</tr>
<tr>
<td>Trial and Error to coordinate</td>
<td></td>
<td>- Why is that number of pieces important?</td>
</tr>
</tbody>
</table>

Fractions Learning Pathways

- **Partitive Fractions Strategies Continuum**

**Please Note:** Mixed, improper and proper fractions should be interspersed throughout fractions teaching and learning so that the students develop flexibility with these early.

"Models" include linear, area, volume, and set representations.

- **Compose and decompose fractions:**
  - Fraction bars
  - Fraction dominoes

- **Add and subtract fractions:**
  - Add and subtract fractions with like denominators:
    - Using models and symbols
  - Add and subtract fractions with unlike denominators:
    - Using models and symbols

- **Unit fractions:**
  - Use unit fractions to name and count fractional amounts
  - Use proportional reasoning to make reasonable estimates

- **Comparing fractions:**
  - Compare fractions with like denominators:
    - Numerators and symbols
  - Generate and recognize equivalent fractions using models and symbols

- **Operations with fractions:**
  - Add and subtract fractions with like denominators:
    - Using models and symbols
  - Add and subtract fractions with unlike denominators:
    - Using models and symbols
**Counting Principles**

### Principles of Counting

- **What skills are involved in counting?**

### Success Criteria:

<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can say the numbers in order.</td>
<td>Counting Sequence:</td>
</tr>
<tr>
<td></td>
<td>- Student can say the sequence of numbers in order (1, 2, 3, 4, 5, …)</td>
</tr>
<tr>
<td>I can touch or point to each object only once.</td>
<td>One-to-one pointing:</td>
</tr>
<tr>
<td></td>
<td>- Student can point at or touch each object he/she is counting only once</td>
</tr>
<tr>
<td>I can say the numbers in order when I touch or point to the objects.</td>
<td>Coordinating:</td>
</tr>
<tr>
<td></td>
<td>- Student can assign a count to each object as he/she points at or touches it</td>
</tr>
<tr>
<td>I can show that the last number counted tells how many.</td>
<td>Cardinality:</td>
</tr>
<tr>
<td></td>
<td>- Student can demonstrate that the final number count represents all the objects in a set, rather than just the last one</td>
</tr>
<tr>
<td>I can show the number in different ways.</td>
<td>Conservation:</td>
</tr>
<tr>
<td></td>
<td>- Student can demonstrate an understanding that a set of objects is the same amount regardless of the configuration or orientation.</td>
</tr>
<tr>
<td></td>
<td>- Students can trust the count, without recounting.</td>
</tr>
<tr>
<td>I can subitize.</td>
<td>Subitzing:</td>
</tr>
<tr>
<td></td>
<td>- Student can identify the number of objects without counting.</td>
</tr>
<tr>
<td></td>
<td>- Students can trust the count, without recounting.</td>
</tr>
</tbody>
</table>

---

**Slides 21 – 28 and 35**

**Continuum of Numeracy Development of Addition and Subtraction**

While watching the videos, identify the strategies the students are using.

---

**Slides 32 - 35**

**Principles of Counting Success Criteria with Student Columns**
Using Videos to Place Students on the Continuum

1. Primary Videos
2. Junior Videos
3. Intermediate Videos

Practice the Process of Using the Continuum
- Anticipate (do the math using different strategies)
- Notice and Name Student Work
- Ask Effective Questions?
- Provide Descriptive Feedback
- Place on Continuum
- Determine Key Ideas (where on the band?)
- Intentional Teacher Moves
Primary Student: Moving from Counting on to Counting on From the Larger Number. Key Idea required is Commutative Property and knowing which number is bigger.

Slides 36 - 39

Primary Student: Subitizing, Number Sequence, Tagging and Co-ordinating

Slides 18 - 24

Primary Student: Key Idea of Hierarchical Inclusion, Cardinality, Part-Whole Relationships.

Slide 25

Primary Student: Subitizing, counting sequence. Needs key idea of organization.

Slides 47 - 48

Primary Student: Key Idea, next step is organization.

Slides 50 - 51

Primary Student: Additive needs to move to Multiplicative Counting

Slides 51 - 53

Practice the Process of Using the Continuum
- Anticipate (do the math using different strategies)
- Notice and Name Student Work
- Ask Effective Questions?
- Provide Descriptive Feedback
- Place on Continuum
- Determine Key Ideas (where on the band?)
- Intentional Teacher Moves
Junior Student: Moving from Counting on to Counting on From the Larger Number. Key Idea required is Commutative Property and knowing which number is bigger.

**Slides 43 - 45**

Junior Student: Moving to Proficiency, being flexible with numbers.

**Slides 27 - 29**

Junior Student: Doubling, Automatic Retrieval. Key Understanding of Unitizing, Part-whole relationships.

**Slides 57 - 58**

Junior Student: Using 10 times, ratio tables. Key Idea of Unitizing, proportional reasoning and place value.

**Slides 28 - 30, 31-32**

Junior Student: Skip counting and Unitizing. Importance of anticipating to push student to multiplicative thinking.

**Slides 57 - 58**

Junior Student: Moving to Proficiency, being flexible with numbers.

**Slides 30 - 33**

Junior Student: Addition Algorithm

**Slides 30 - 33**

Junior Student: Change Unknown Question

**Slides 30 - 33**

Junior Student: Subtraction Algorithm

**Slides 44 - 45**

Junior Student: Splitting and Automatic Retrieval. Key ideas of Place Value and hierarchical inclusion.

**Slides 50 - 51**

Junior Student: Doubling, Automatic Retrieval. Key Understanding of Unitizing, Part-whole relationships.

**Slides 30 - 33**

Junior Student: Represent empty groups and fair sharing. Common Error is the using the distributive property incorrectly.

**Slides 31-32**

Junior Student: Seeing SLIDES 59 and 60 Repeated Subtraction, move thinking forward without drawing boxes.

**Slides 59 - 60**

Junior Student: Skip counting and Unitizing. Importance of anticipating to push student to multiplicative thinking.

**Slides 57 - 58**

Junior Student: Represent empty groups and fair sharing. Common Error is the using the distributive property incorrectly.

**Slides 31-32**
Intermediate Videos

Focusing on the Fundamentals of Math – Day 1
CONNECTING OUR WORK TO THE FUNDAMENTALS

Intermediate Student: Counting on / Counting Back and Taking Jumps of 10
Slides 23 - 29

Intermediate Students: Moving from Counting on to Counting on From the Larger Number.
Slides 48 - 51

Intermediate Student: Part - Whole Relationships, limited understanding of Place Value
Slides 38 - 39

Intermediate Student: Key Ideas of Place value and associative property. Used splitting strategy incorrectly. Student lost sense of the problem.
Slides 41 - 42

Focusing on the Fundamentals of Mathematics - Day 2
CONNECTING OUR WORK TO THE FUNDAMENTALS: DAY 2

Intermediate Student: Representing Empty Groups and Fair Sharing. Key Idea - unitizing the 5
Slides 64 - 66

Intermediate Student: automatic retrieval, using formulas, partial products.
Slides 73-74

Focusing on the Fundamentals of Mathematics – Day 4
CONNECTING OUR WORK TO THE FUNDAMENTALS: DAY 4

Slides 35-36

Slides 37-39

Intermediate Student: Using alternative or standard algorithm, representing empty groups and fair sharing. Using familiar facts.
Slides 40-42

Click on the blue boxes to see videos

Practice the Process of Using the Continuum
- Anticipate (do the math using different strategies)
- Notice and Name Student Work
- Ask Effective Questions?
- Provide Descriptive Feedback
- Place on Continuum
- Determine Key Ideas (where on the band?)
- Intentional Teacher Moves
"Noticing and Naming the Learning": The Link to Learning Goals and Success Criteria

As discussed in Chapter 4 of Growing Success, "Assessment for Learning and Assessment as Learning" (Ontario Ministry of Education, 2010), teachers and students develop a common understanding of learning goals – what is being learned – and of success criteria – what successful attainment of the learning goals looks like (p. 33). In Kindergarten, educators and children also develop a shared understanding of what is being learned, of where the children are in their learning, and of where they are going next. The children come to understand what growth in their learning "looks like". "Noticing and naming the learning" is a strategy employed in Kindergarten that provides the link to more formal approaches in the use of learning goals and success criteria in the later grades.

The range of what we think and do is limited by what we fail to notice. And because we fail to notice that we fail to notice, there is little we can do to change until we notice how failing to notice shapes our thoughts and deeds.  

(Goleman, 1985)
Focusing on the Fundamentals Of Math

Anticipation Guide
Anticipation Guide Answer Key
FFM Teacher Guide
FFM Parent Guide
Doing Mathematics with Your Child at Home
Memorization vs Automaticity

Videos: Five Fundamental Math Concepts and Skills
Working with Numbers
Recognizing and Applying Understanding of Number Properties
Mastering Math Facts
Developing Mental Skills
Developing Proficiency with Operations

Purposeful Planning
5 Practices
Curriculum
Effective Guides to Mathematics Instruction
Know Your Learner
Lesson Planning / 5 Day Plans
Minilessons

Continua
Addition and Subtraction Continuum
Multiplication and Division Continuum
Fractions Continua
Subitizing Learning Trajectory (Continuum)
Comparing Learning Trajectory (Continuum)
The Process of Using the Continua

Structure of the Problem
Addition and Subtraction
Multiplication and Division

Focusing on the Fundamentals of Math
A strong understanding of math fundamentals is one of the best ways to prepare students for success, now and in the future. As learners progress through elementary school, they will develop their ability to think mathematically, learn about different concepts and relationships, and apply their knowledge. Ontario’s focus on the fundamentals of math will help Ontario students achieve higher math results and will better position them for success in daily living and the jobs of tomorrow. It will also help them connect what they are learning in school to real life, and solve everyday problems.

Math – The Learning Exchange
Big Ideas in Math

Topics on slides include: Algebraic Reasoning, Conjectures, Justifying and Proving, Meaning of Equal Sign, etc. (Slides 20 – 33, 40 -50, 52-55, 57-58)

Placemat
Fairy Tale Problem (Slides 54 - 55)
Fairy Tale Problem Sample Work

Continuum of Curriculum Expectations

Slides 22 – 42
Fractions Learning Pathway
Fraction Planning Template
Fractions Key Ring
Fractions Continuum Slides 46-90
Partitive Fractions Strategies Continuum
Slides 13 – 14
Fractions Matching Activity

Subitizing Continuum PowerPoint
What is subitizing?
Learning Trajectories
Using the Continuum
Games
Resources
CASMT Model
Stages of Subitizing Instructional Guide

Mathematical Processes
Mathematical Processes Videos
Problem Solving (Slide 19)
Problem Solving Success Criteria
Problem Solving Success Criteria Sample 1
Problem Solving Success Criteria Sample 2
Beginning to Problem Solve with “I Notice, I Wonder”

Slides 18 – 23
Task on Slide 19
Continuum of Curriculum Expectations

Slides 10 -12
Paper Folding Task Instructions
Slide 17
Continuum of Curriculum Expectations
EOSDN – Kindergarten / Grade 1 Inquiry

- Subitizing
- Learning Trajectories / Continuums
- Using the Continuum
- Worthwhile Mathematical Tasks
- Games
- Resources
- CASMT Model
Subitizing

Connection to Other Number Concepts

Representing Numbers

Number Relationships

Number Operations

Part Whole Relationships

Quantity and Number

Image Placement

Curriculum Connections

Estimation and Quantity

Unitizing

Comparing Number

Slide Decks

Number Talks

Resources

Two Types of Subitizing

Perceptual Subitizing

Conceptual Subitizing

Slides 6 to 9

Learning Trajectories / Continuums

Progression of Early Number and Counting

Back to Big Ideas in Math

Back to Kinder / GR 1 Inquiry
<table>
<thead>
<tr>
<th>Connection to Other Number Concepts</th>
<th>EOSDN Day 1 – Slide 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representing Numbers</td>
<td>EOSDN Day 1 – Slides 11 to 13</td>
</tr>
<tr>
<td>Number Relationships</td>
<td>EOSDN Day 1 – Slides 15 to 20</td>
</tr>
<tr>
<td>Number Operations</td>
<td>EOSDN Day 2 – Slides 52 to 61, Tiny Polka Dots</td>
</tr>
<tr>
<td>Part Whole Relationships</td>
<td>EOSDN Day 1 – Slide 23</td>
</tr>
<tr>
<td>Quantity and Number</td>
<td>EOSDN Day 1 – Slides 31 to 33</td>
</tr>
<tr>
<td>Image Placement</td>
<td>EOSDN Day 1 – Slides 39 to 44, EOSDN Day 2 – Slides 7 and 8</td>
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<tr>
<td>Curriculum Connections</td>
<td>EOSDN Day 1 – Slide 4, Paying Attention to Subitizing</td>
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<tr>
<td>Estimation and Quantity</td>
<td>EOSDN Day 1 – Slides 25 to 30</td>
</tr>
<tr>
<td>Unitizing</td>
<td>EOSDN Day 2 – Slides 23 to 24, 46 to 48</td>
</tr>
<tr>
<td>Comparing Number</td>
<td>Trajectory and Continuum</td>
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</table>
Subitizing Trajectory from Clements and Sarama

<table>
<thead>
<tr>
<th>Key Characteristics</th>
<th>Subitizing Trajectory from Clements and Sarama</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage of Development</td>
<td>Pre-Explicit Number</td>
</tr>
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</tbody>
</table>

Notes

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Continuum of Learning for Comparing Number

Continuum of Learning for Comparing Number

Learning Trajectory of Comparing Numbers

Website: www.LearningTrajectories.org
### Subitizing Learning Trajectory

**Subitizing Trajectory from Clements and Sarama**

#### Stages of Subitizing Instructional Guide
- Includes 48 UCDSB student videos by stage and learning experiences to move student along the continuum / trajectory.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Pre-Explicit Number</th>
<th>Small Collection Namer</th>
<th>Maker of Small Connections</th>
<th>Perceptual Subitize to 2</th>
<th>Perceptual Subitize to 5</th>
<th>Perceptual Subitize to 10</th>
<th>Conceptual Subitize to 20</th>
<th>Conceptual Subitize with Place Value and Skip Counting</th>
<th>Conceptual Subitize with Place Value and Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notice</td>
<td>Characteristics: Notice the difference in size of 1 and 2, and the presence of subitizing (non-explicit) knowledge of number.</td>
<td>Shown and named sets of 1 and 2, and sometimes 3 and 4. Notice the difference in size of 1 and 2, and the presence of subitizing (non-explicit) knowledge of number.</td>
<td>Initially uses one of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
<td>Initially uses any of fewer than 6 items in sets as a unit.</td>
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<tr>
<td>Notes</td>
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</tbody>
</table>

#### Using the Continuum

**Stage: Small Collection Namer Video(s):**

**Small Collection Namer Video 1**

**Small Collection Namer Video 2**

- What intentional games might you have the student(s) play?
- Explain why you chose these games?
- What number talk/string might you use?
- Explain why you chose this number talk/string?
- What mini-lesson might you provide?
- Explain why you chose this mini-lesson?
- What provocations at your workstations might you consider?
- Why these provocations?
- What learning will take place through the four frames of the Kindergarten Program or cross-curricularly in Grade 1?

**Learning Experiences and Reasons**

- Number String
  - Use the same number in different configurations.
- Number Talk
  - Splits of 3, 4, 5 when ready
- What doesn't belong...3, 8, playing card, dry erase dice – document student thinking (students using hand signals)
- What mini-lesson might you provide?
- What provocations at your workstations might you consider?
- Why these provocations?
- What learning will take place through the four frames of the Kindergarten Program or cross-curricularly in Grade 1?

**Back to Big Ideas in Math**
Using the Continuum

Practice the Process of Using the Continuum
- Anticipate (do the math using different strategies)
- Notice and Name Student Work
- Ask Effective Questions
- Provide Descriptive Feedback
- Place on Continuum
- Determine Key Ideas (where on the band?)
- Intentional Teacher Moves

Claire:
**ESODN Day 1 – Slides 51 and 52**
**Student Video**

**EOSDN Day 2 – Slides 38 – 40**
**Student Video**

Millie - Rose:
**EOSDN Day 1 – Slides 47 and 48**
**Student Video**

**EOSDN Day 2 – Slides 29 - 33**
**Student Video**

Quinn:
**EOSDN Day 1 – Slides 49 and 50**
**Student Video**

**EOSDN Day 2 – Slides 33 - 37**
**Student Video**

**EOSDN Day 3 – Slides 31 - 25**
**Student Video 1**

**Student Video 2**

Other UCDSB Students:
**EOSDN Day 3 – Slides 36 - 42**
**Student Video 1**

**Student Video 2**

---

### Stage: Small Collection Name

**Video(s):**
- **Small Collection Name Video 1**
- **Small Collection Name Video 2**

<table>
<thead>
<tr>
<th>Learning Experiences</th>
<th>Learning Experiences and Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>What intentional games might you have the student(s) play?</td>
<td><em>Learningexperiencesexamples.org</em> Build a Plane.</td>
</tr>
<tr>
<td>Explain why you chose these games?</td>
<td>Circle Game (roll the dice, make it on the plate, partner verifies, partner puts on their plate.)</td>
</tr>
<tr>
<td>What number talk/string might you use?</td>
<td>What is next for:</td>
</tr>
<tr>
<td>Explain why you chose this number talk/string</td>
<td>Dittogram on page 10.</td>
</tr>
<tr>
<td>What mini-lesson might you provide?</td>
<td>Memory Game 1-4</td>
</tr>
<tr>
<td>Explain why you chose this mini-lesson?</td>
<td>Make and Test: rolling bear around the classroom (in group).</td>
</tr>
<tr>
<td>What provocations at your workstations might you consider?</td>
<td>To give students experiences with hands on activities during their thinking with a peer: practice recognizing different representations of numbers.</td>
</tr>
<tr>
<td>Why these provocations?</td>
<td>Increase student exposure to a variety of representations.</td>
</tr>
<tr>
<td>What learning will take place through the four frames of the Kindergarten Program or cross curricularly in Grade 1?</td>
<td>Read Aloud Ten Bears by Donald Crews up to Number 4.</td>
</tr>
</tbody>
</table>

Stages of Subitizing Instructional Guide. Includes 48 UCDSB student videos by stage and learning experiences to move student along the continuum / trajectory.
## Games

### Slides 21, 33, 35

**EOSDN – Day 1**
North Greeneville Municipal Centre
February 13, 2019

### Slides 77 and 78

**EOSDN – Day 2**
Grenville Mutual Insurance Building
April 3, 2019

### Slide 39

**EOSDN – Day 3**
Grenville Mutual Insurance Building
May 10, 2019

### Games in English

<table>
<thead>
<tr>
<th>Game</th>
<th>Strategies</th>
<th>Where Referenced</th>
<th>Materials</th>
<th>Group Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Fish 10</td>
<td>strategy of using the relationship of ten within the relationship of ten and subtraction</td>
<td>Video 2.9 (page 36) Video 2.11 (page 42) Video 2.12 (page 42)</td>
<td>an捅stack of 10 of playing cards with the tens, ones, queens, and kings removed (from the game of snap)</td>
<td>2 to 5 players</td>
</tr>
<tr>
<td>Make 10 Concentration</td>
<td>strategy of using the ten within the relationship of ten within the relationship of ten and subtraction</td>
<td>Video 2.9 (page 36) Video 2.11 (page 42) Video 2.12 (page 42)</td>
<td>an捅stack of 10 of playing cards with the tens, ones, queens, and kings removed (from the game of snap)</td>
<td>2 players</td>
</tr>
<tr>
<td>Steal the Beans</td>
<td>strategy of using the ten within the relationship of ten within the relationship of ten and subtraction</td>
<td>Video 2.9 (page 36) Video 2.11 (page 42) Video 2.12 (page 42)</td>
<td>an捅stack of 10 of playing cards with the tens, ones, queens, and kings removed (from the game of snap)</td>
<td>2 to 4 players</td>
</tr>
<tr>
<td>Make 10 and Add to the 10</td>
<td>strategy of using the ten within the relationship of ten within the relationship of ten and subtraction</td>
<td>Video 2.9 (page 36)</td>
<td>a board game (optional)</td>
<td>2 to 4 groups</td>
</tr>
</tbody>
</table>

### Games in French

### Summary of games in What to Look For

- **Tiny Polka Dot**
  Games in the Classroom
  - Fill the Tower Alternate Versions

- **Tug of War**
Resources

Steve Wyborney – 20 days of Numeracy, Splats, Cube Conversations, Esti-mysteries, Estimation Clipboard
Math Before Bed – visual prompts
MathforLove – lessons and activities (creator of Tiny Polka Dots)
Daily Routines, Teach at the Speed of Light – lots of images and activities for number talks
Math Tools

Mathies Learning Tools

Rekenrek

Use the Rekenrek to build understanding for counting, addition and 10. Use the annotation feature to write notes.

Additional features (v 1.3.1 Dec

- minor bug fixes
- correct links to support pages
- addresses recent changes

Learning to Think Mathematically with the Rekenrek

A Resource for Teachers, A Tool for Young Children

Jeff Frykholm, Ph.D.
Math Talk

Splat!

Same But Different Math

3(x+1) = 15

WODB

Which would you rather purchase jeans from?

Store A

Store B

30% off one pair

$30 dollars off

www.mathbeforebed.com

Steve Wybourny, Cube Conversations

Slides 14 - 22

Back to Subitizing Tasks
Collaborative Analysis of Students’ Math Thinking (CASMT)

Record the student strengths and needs on separate post-it notes.

- What mathematical thinking is evident? (RED)
- What partial/transitional understandings are evident? (GREEN)
- What connections can you make to the strengths and/or needs (e.g., listed in the IEP or Student at a Glance)? (BLUE)
- What wonderings do you have? (PURPLE)
- What next steps are you considering for this student? (ORANGE)

Consider the following questions...

- What mathematical thinking is evident? (RED)
- What partial/transitional understandings are evident? (GREEN)
- What connections can you make to the strengths and/or needs (e.g., listed in the IEP)? (BLUE)
- What wonderings do you have? (PURPLE)
- What next steps are you considering for this student? (ORANGE)

You may wish to refer to YCDSB Supporting Students with Learning Disabilities in Mathematics resource document.
EOSDN Slide Shows

EOSDN – Day 1
North Grenville Municipal Centre
February 13, 2019

EOSDN – Day 2
Grenville Mutual Insurance Building
April 3, 2019

EOSDN – Day 3
Grenville Mutual Insurance Building
May 10, 2019